Find all the real numbers satisfying the following inequalities. Write the solution using intervals and represent it in the real line.

(a) $x^4 - 16 \geq 0$.

(b) $x^{\frac{2}{3}} - x^{\frac{1}{3}} \leq 0$.

(c) $\dfrac{x+1}{x-1} \geq \dfrac{x}{x-1}$.

(d) $x^2 - 4 \geq 0$

(e) $2x + 5 < 3x - 4$

(f)
$$\frac{3x+2}{x-3} < 2.$$

Exercise 1 Solve the following inequalities, clearly indicating all intermediate steps. Use the interval notation to give your answer:

- $\dfrac{x^2 - x}{x^2 - 4} \geq 0$

- $\dfrac{x}{2x+2} - \dfrac{3}{x} \leq 4 - \dfrac{5}{x+1}$

- $x^2 - 9 \leq 0$

Exercise 2 Solve the following equations and inequalities, clearly indicating all intermediate steps. Use the interval notation to give your answer:

- $|3x + 2| = |5x - 1|$

- $-2|3x - 2| - 3 \leq 8$

- $3|x - 1| + 2x \geq 3x - 2$

Exercise 3 Graph the following equations (give values to the x and y to find points (a, b) which satisfy the equation). Which of them represent functions?

- $y = 3x^2 - 1$

- $5y + 3x = 1$

- $y^2 = 3x - 1$

- $|x| - 2|y| = 4$

- $y = \begin{cases} 2x & \text{if } x < 1 \\ x^2 - 3 & \text{if } 1 \leq x < 3 \\ 6 - 2x & \text{if } x \geq 3 \end{cases}$

Exercise 6 Decide whether the following functions are even, odd, or neither:

- $f(x) = 3x^2 - 1$
- $g(x) = x^3 + 2x$
- $h(x) = 3x^2 - 1 + x$
- $t(x) = x^3 + 2x - 1$

Challenging question: do you see any general rule applying to when a polynomial of even/odd degree will be an even/odd function?

Exercise 7 Given the points $(-3, 5)$ and $(2, -7)$:

1. Find the distance between them

2. Find the equation of the line which passes through them

3. Find the equation of the line perpendicular to the previous one which passes through the point $(1, 0)$

4. Find the equation of a line parallel to the first one

Exercise 8 Given the parabola $y = 2x^2 - 6x + 4$, find:

1. interception with the axis

2. vertex

3. domain

4. range

5. axis of symmetry

Sketch its graph.